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State Examinations Commission
Cornamaddy
Athlone
Co. Westmeath

Dear Sir/Madam,

Re: Errors in Project Maths sample paper

I am writing regarding your recent document ‘Leaving Certificate Examination, 2012, Sample Paper: Mathematics (Project Maths — Phase 1), Paper 2, Ordinary Level’¹. In general, I think this and other Project Maths papers are doing a good job of testing whether candidates have developed appropriate intuition in many areas of mathematics, particularly applied maths. However, there are a couple of places where this has been achieved at the cost of rigour and correctness. While this document is only a sample paper, I am hoping to hear your views on this.

To be specific, two parts of q.1 of this 2012 sample are incorrect. The question concerns four sets of data, whose size, mean and standard deviation are given in the question as follows:

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
size (<i>N</i>)	1000	100	100	10
mean (μ)	10	100	1000	100
standard deviation (σ)	20	30	20	10

No further information is given about the data-sets. Part (d) of the question asks the candidate to fill in the blank in this sentence, where the bold is in the original:

The set that **must** contain some negative numbers is set _____.

The intended answer is probably ‘set *A*’. However, it is possible to construct a 1000-point data-set with $\mu = 10$ and $\sigma = 20$ where every element is non-negative. One way is:

$$a_1 = a_2 = \dots = a_{999} \approx 9.36 \geq 0; \quad a_{1000} \approx 642 \geq 0.$$

Sets *B*, *C*, and *D* more obviously need contain no negative values. One could make the claim that ‘sensible’ 1000-point sets with $\mu = 10$ and $\sigma = 20$ would contain negative numbers, but the question includes the emphasised word ‘**must**’, and that

¹http://www.examinations.ie/schools/Project_Maths_Phase1_P2_Ordinary_Level.pdf

is an absolute word within mathematics. This part of the question therefore has no correct answer.

Part (e) is also wrong, in two ways. It asks the candidate to complete the following sentence.

If the four sets are combined, the median is most likely to be a value in set _____.

One problem is that the combined data-set would contain 1210 points, which is an even number. The conventional definition of median for an even-sized set is the arithmetic mean of its two most-central points. The median would therefore not necessarily be an element of any of the sets.

Suppose for the sake of argument that this error were to be corrected, for example by making set D have $N = 11$. The more basic problem remains: It is meaningless to ask a question about what is ‘most likely’ in this context. The question posits four data sets. It does not say that they are samples drawn from some distribution(s). They are fixed, albeit unspecified, data-sets. The median value belongs to (at least) one of the sets, but there is no probability space, so one cannot ask for the probability of the median value belonging to, for example, set A . This part of the question is ill-posed.

With both these errors, one can ‘see what the question is getting at and what the examiner wants’, but this is a fundamentally wrong approach for a mathematics candidate to be forced to take in an exam. Questions ought to be unambiguous and, most importantly, fully correct and rigorous.

The above points stand on their own merits, regardless of who is making them. For the sake of completeness, though, my background is as follows. I was awarded a first-class degree in mathematics from Oxford, UK, and then a DPhil in statistical methods within computer vision, also from Oxford. For the past ten years I have been employed as a quantitative researcher by a financial trading company, using statistics and probability heavily. (Having raised the topic of my employment: this letter is a personal one and nothing about the views of my employer should be inferred from it.) My interest in the mathematics examination process has been encouraged over the past several years, as I have been volunteering as a tutor in a homework club, working with leaving-cert maths students.

I would be grateful for a reply which addresses the substantive mathematical points above, ideally from somebody involved in the setting of SEC mathematics papers.

Thank you for your time.

Yours faithfully,

Ben North